Physics of varying spacetime dimensions

Moorea, PACIFIC 2024

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Cao, Lu, TM SciPost Phys. Core 7, 055 (2024)

Q:

What’s the 2D version of a sphere?

sphere

2D

3D

Circle sphere

2D 3D

Circle

sphere

Q: What's the 4D version of a sphere?

2D

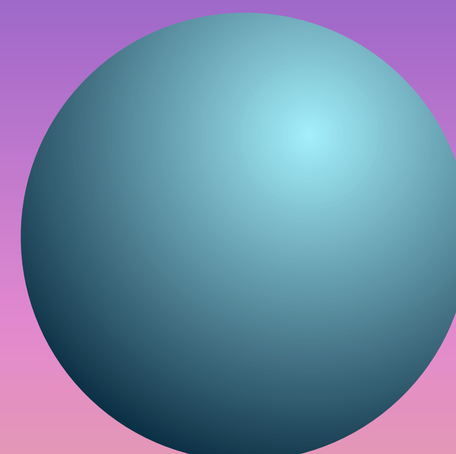
3D

4D

Circle

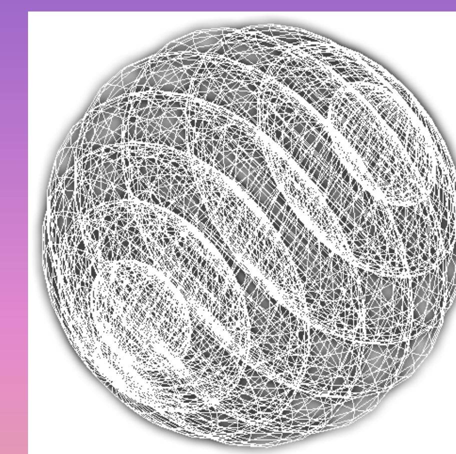
2D

sphere

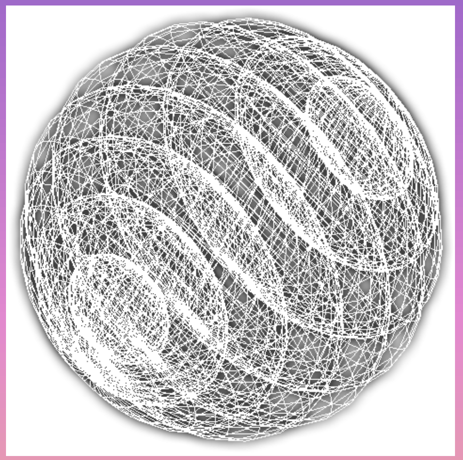


3D

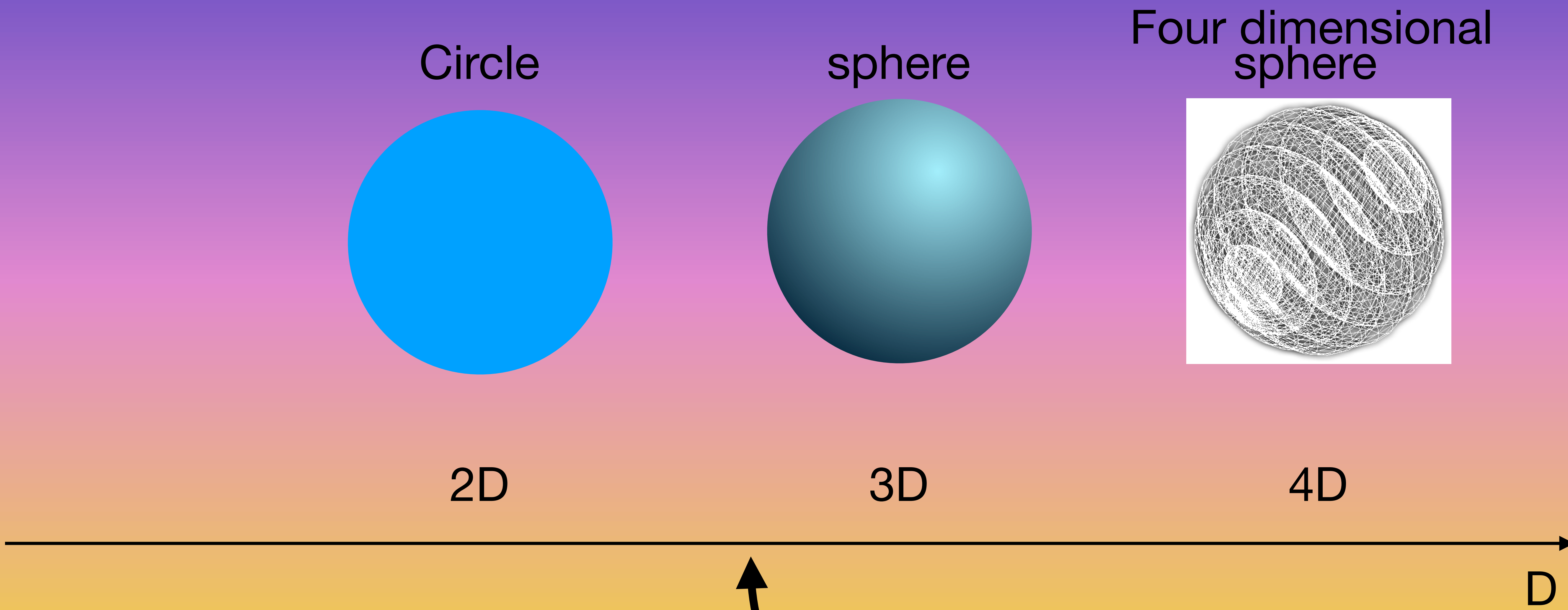
Four dimensional sphere



4D

Circle	sphere	Four dimensional sphere
		
2D	3D	4D

Q: What's the 2.7434 D version of a sphere?

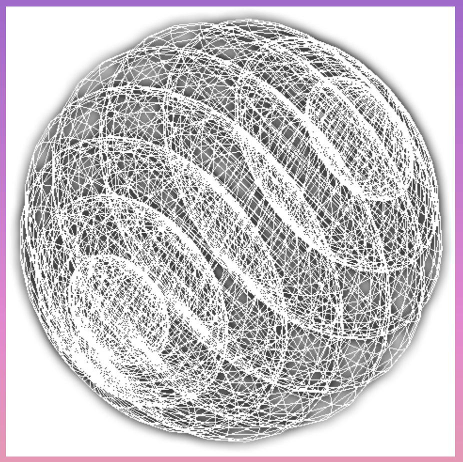


Q: What's the 2.7434 D version of a sphere?

A: A 2.7434 dimensional sphere

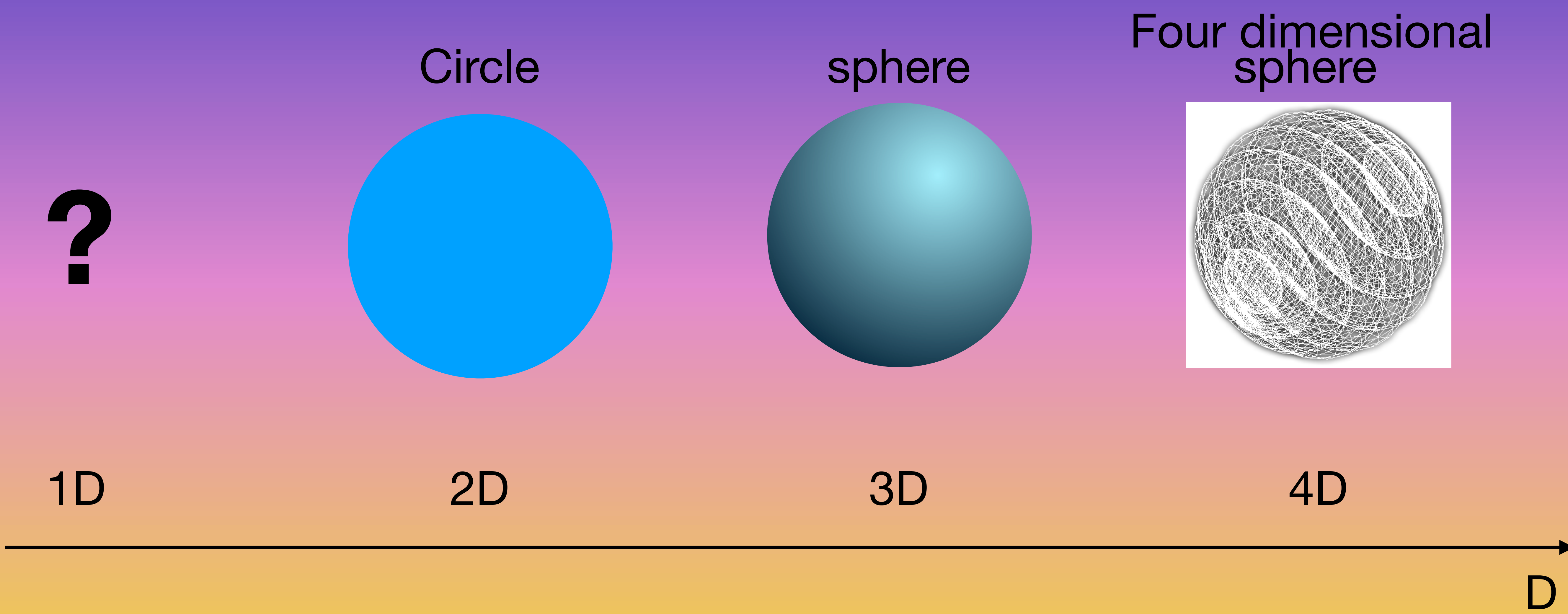
Q:

What's the 1 D version of a sphere?

?	Circle	sphere	Four dimensional sphere
			
1D	2D	3D	4D

D

Q: What’s the 1 D version of a sphere?



A: This talk

Q: Why do we care?

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A1: We here are all dimension ninjas

We think about higher/lower dimensions for all variety of phenomenological and theoretical reasons

We think about 4-e non-integer dimensions to do calculations (most of the time setting e->0 at the end, but not always, e.g. Wilson Fisher fixed point)

Do 1/N expansions

....

Q: Why do we care?

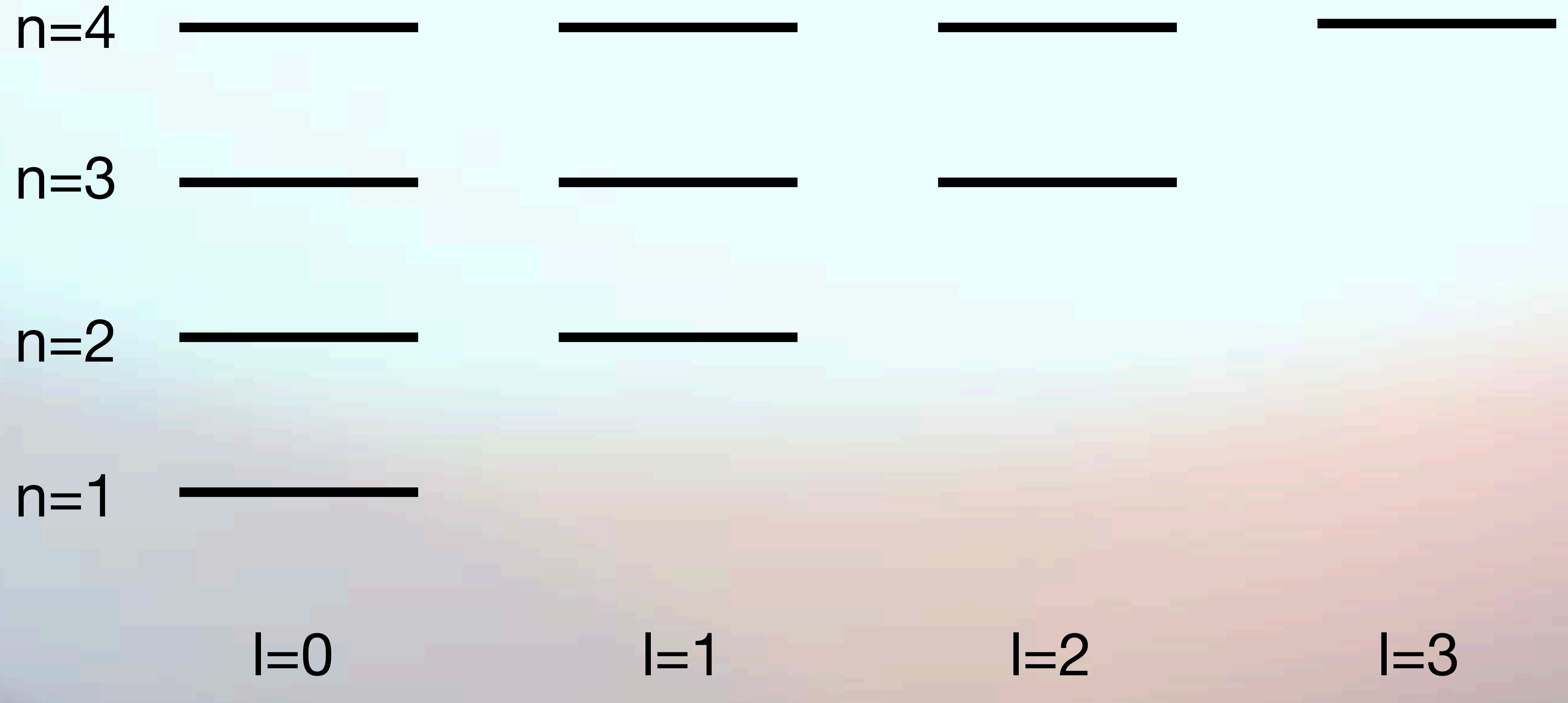
A2: Symmetry is the main tool we have to understand anything about QFT

Lets go back 100 years..

Degeneracy in QM

Quantum mechanics begin when the spectrum of the Hydrogen atom was calculated, circa 1926 (Heisenberg, Pauli, Schrödinger)

It exhibited a striking feature



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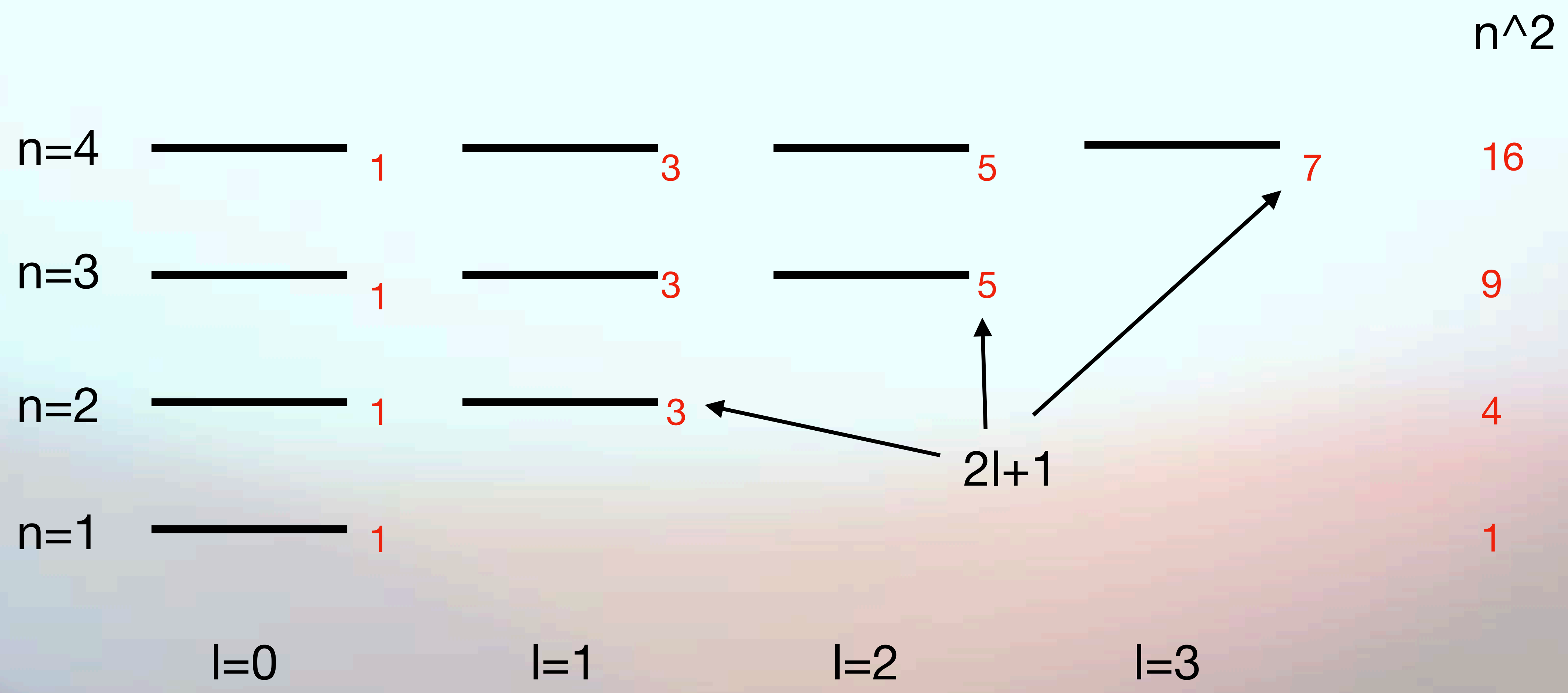

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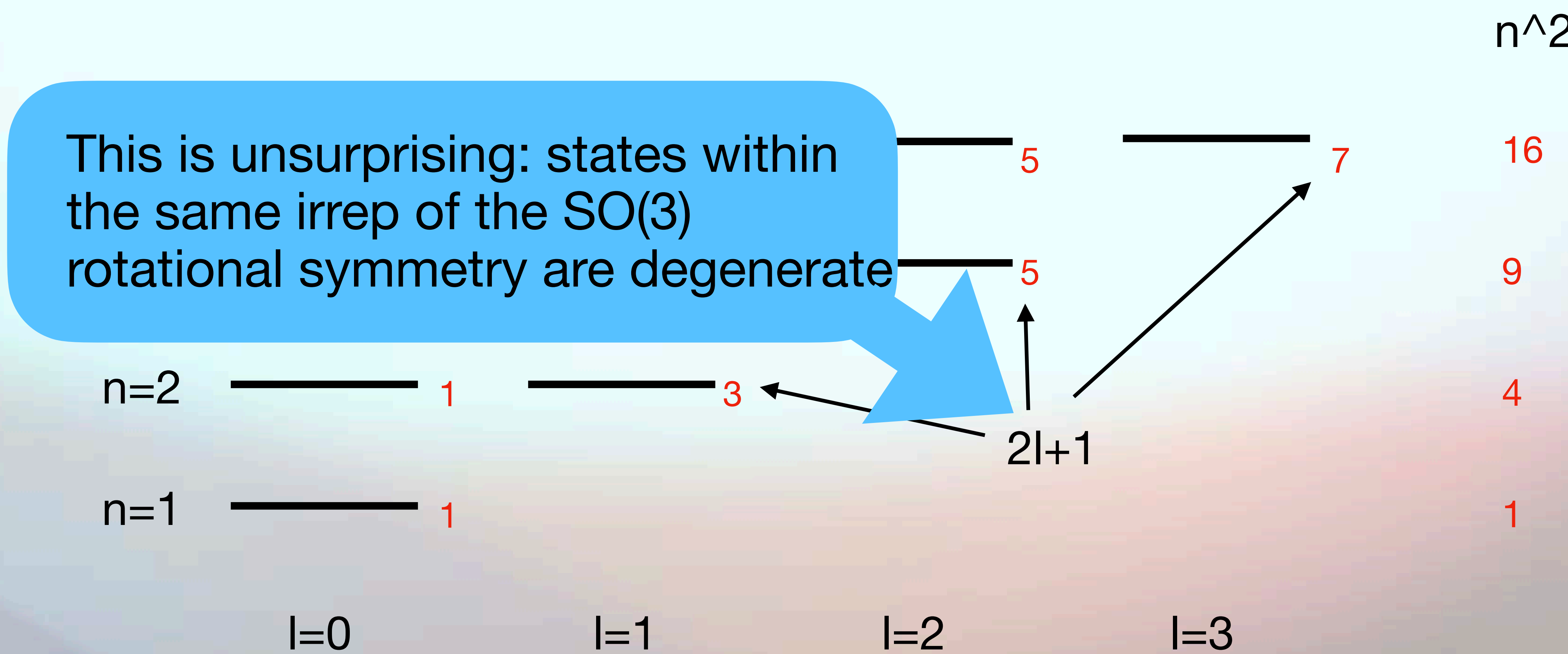

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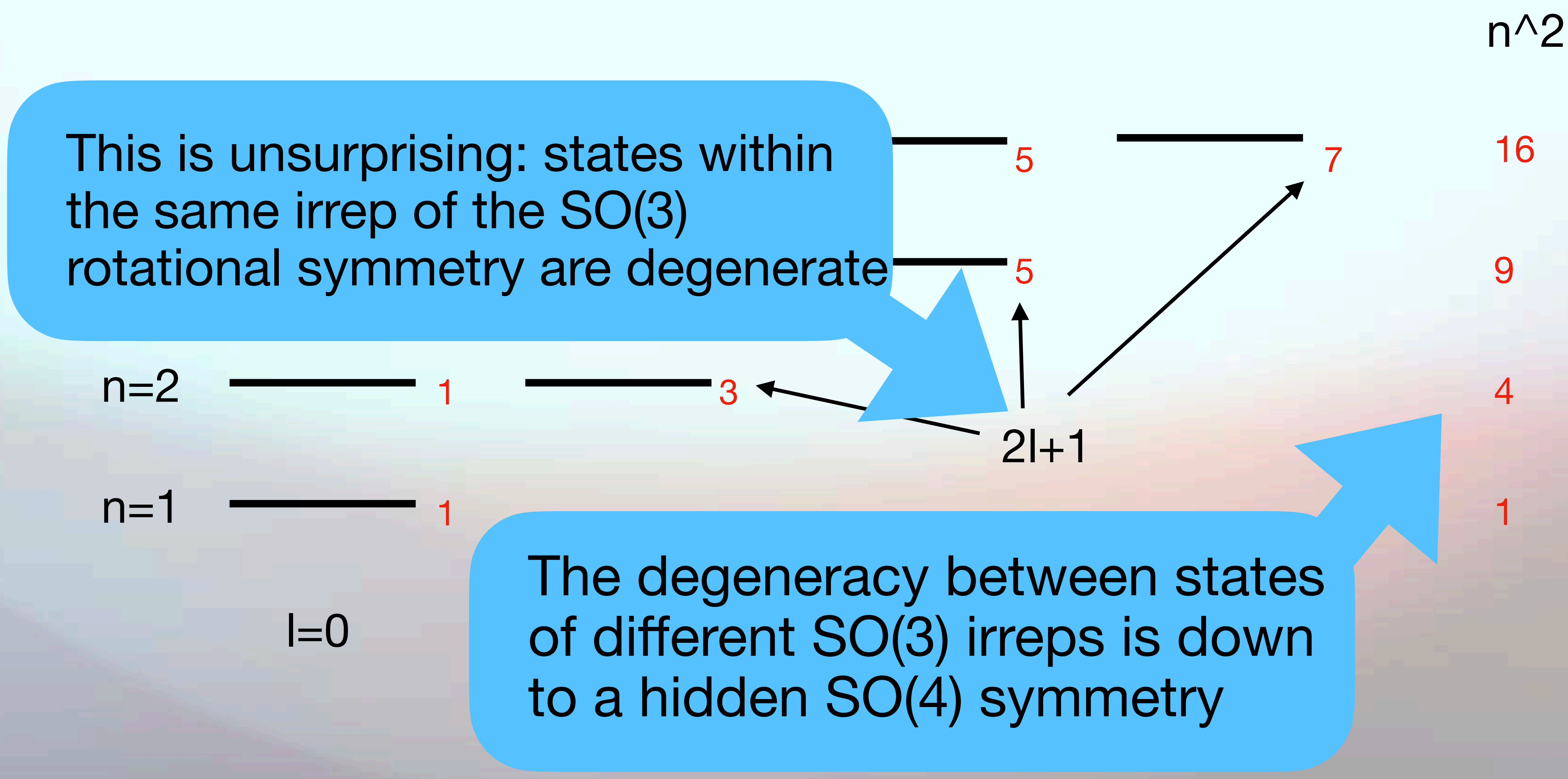
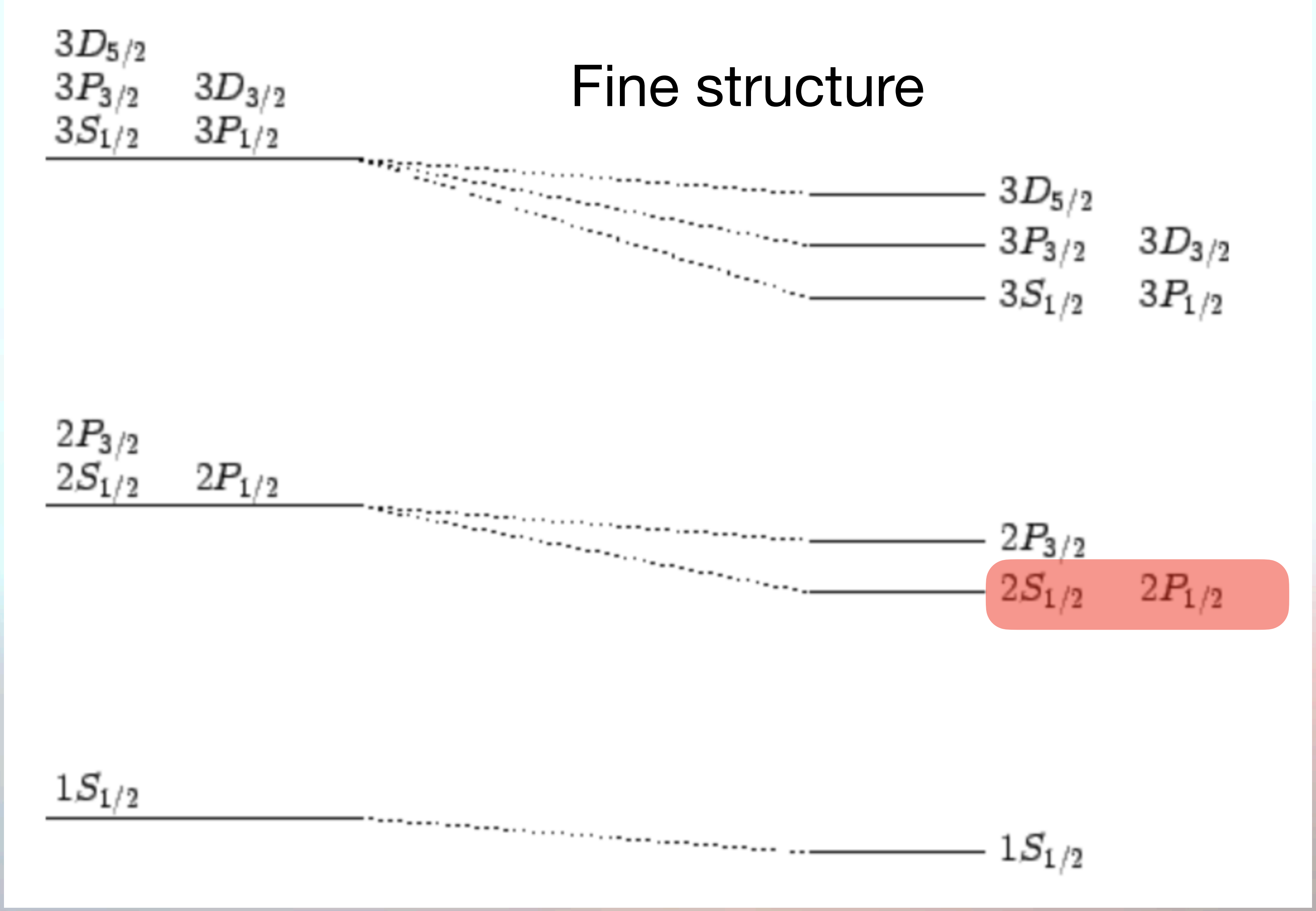


Degeneracy in QM

The Lamb shift played a central role in the development of field theory



Level n degeneracies in first H model

Degeneracies removed by small perturbations

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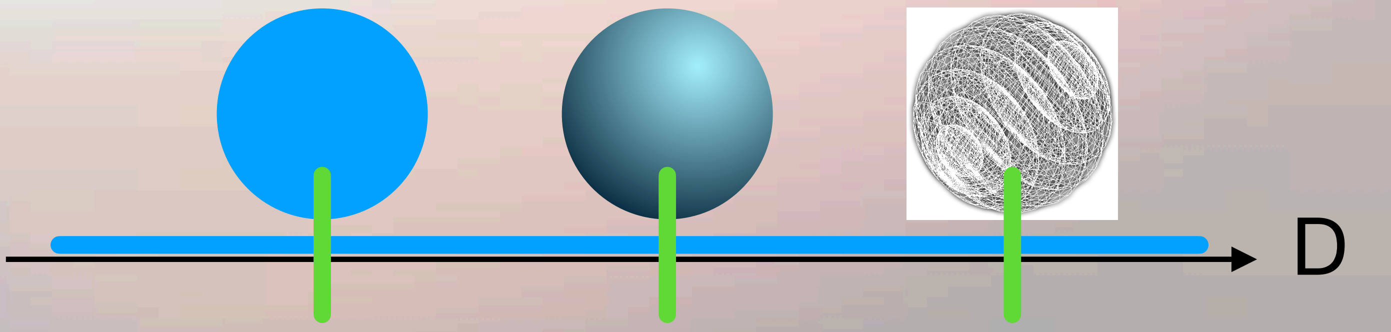
Unitary QFT

Non-unitary QFT



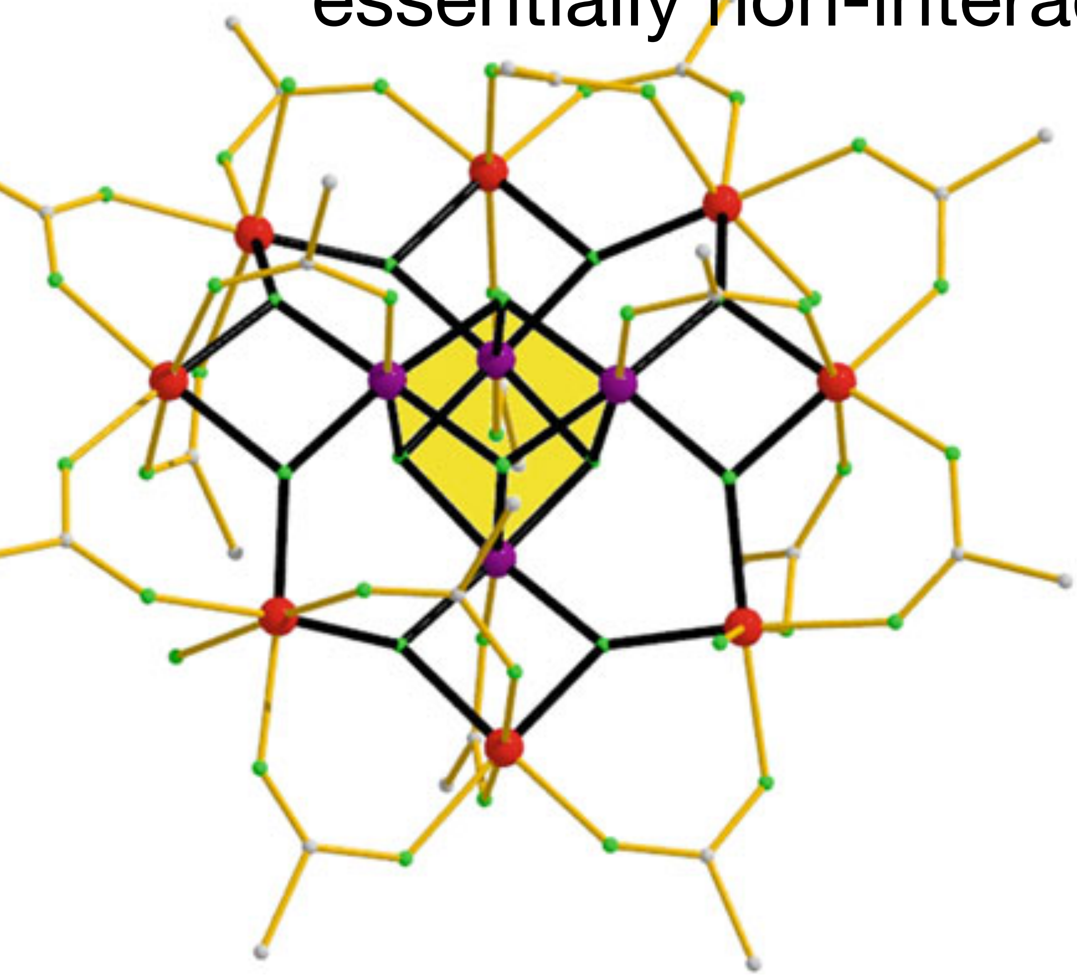
O(N) for non-integer N: If the theory is a CFT, it is non-unitary

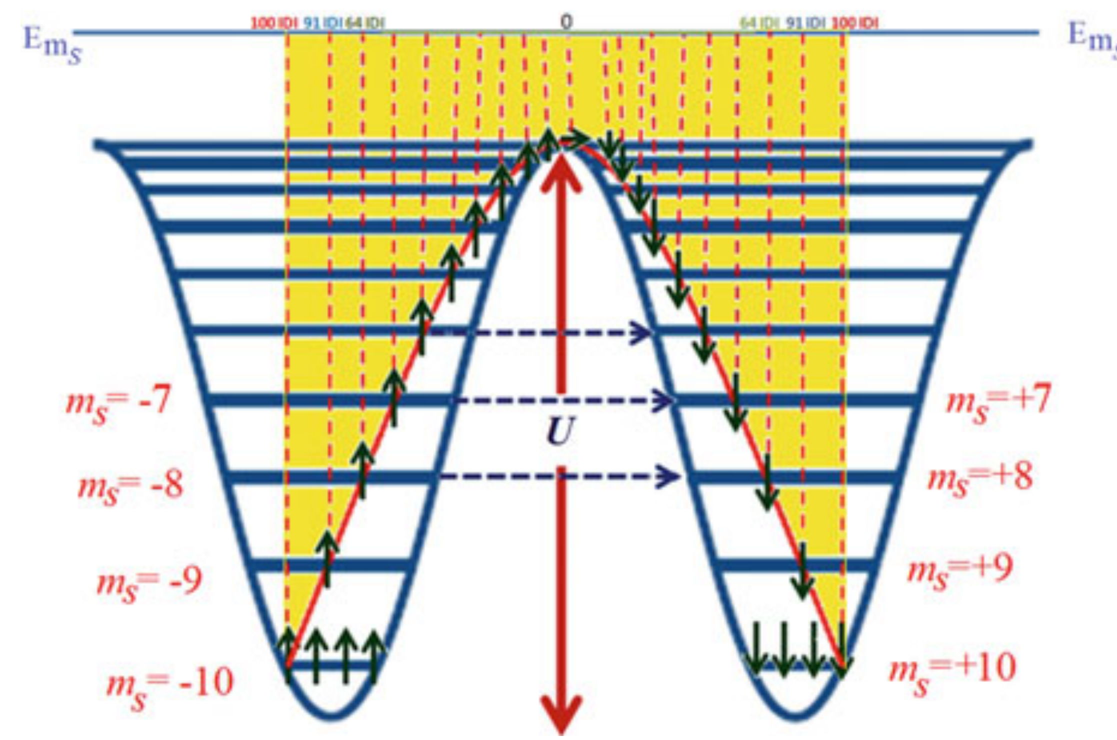
[Binder and Rychkov 1911.07895]

Degeneracy in QM

Macroscopic **changes**

Single molecule magnets are molecular crystals with essentially non-interacting spins at centre of each molecule





$$H = -U \sum_i (S_i^z)^2$$

Degeneracy in QM

Macroscopic changes

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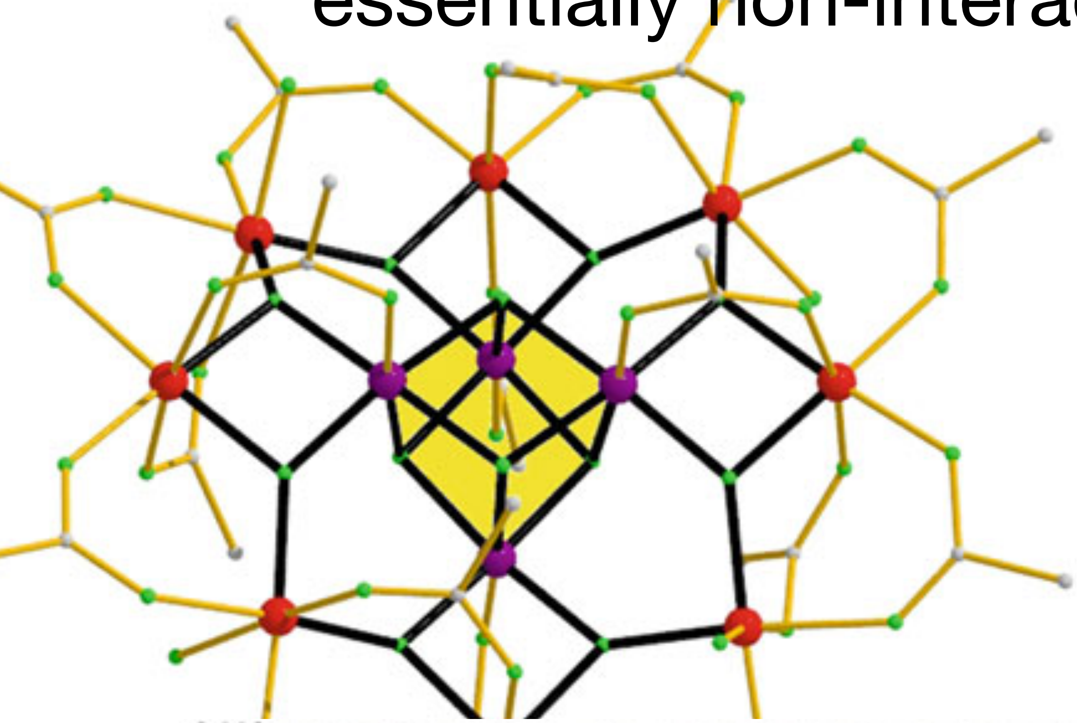
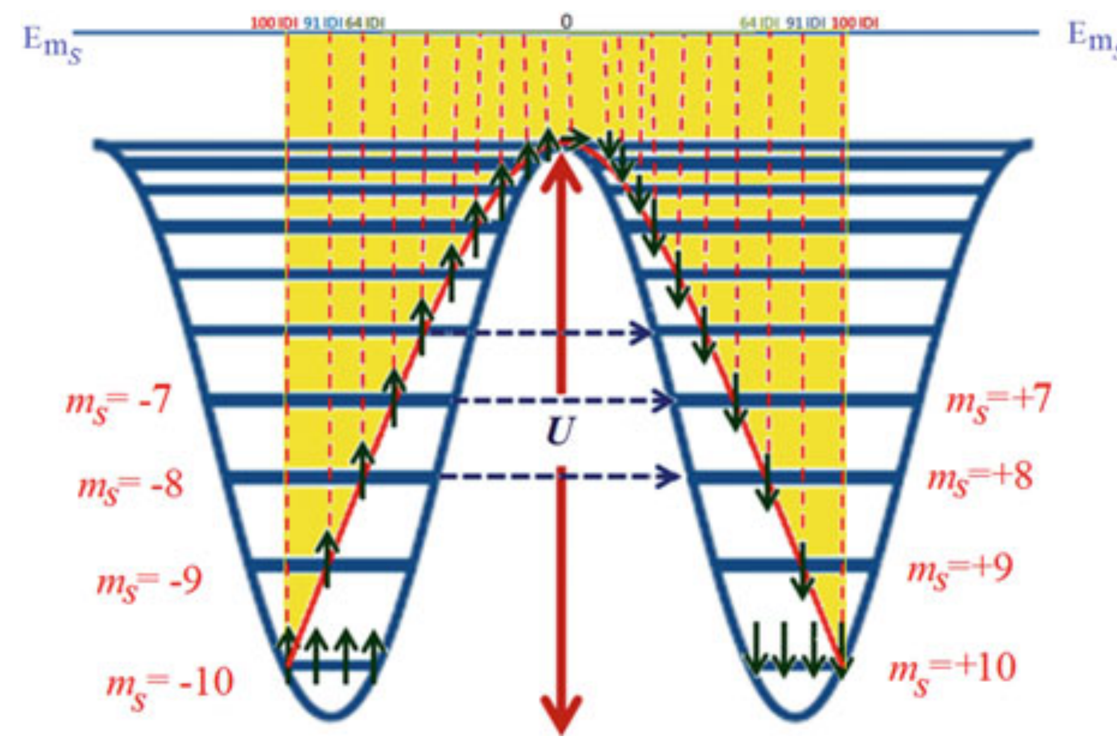
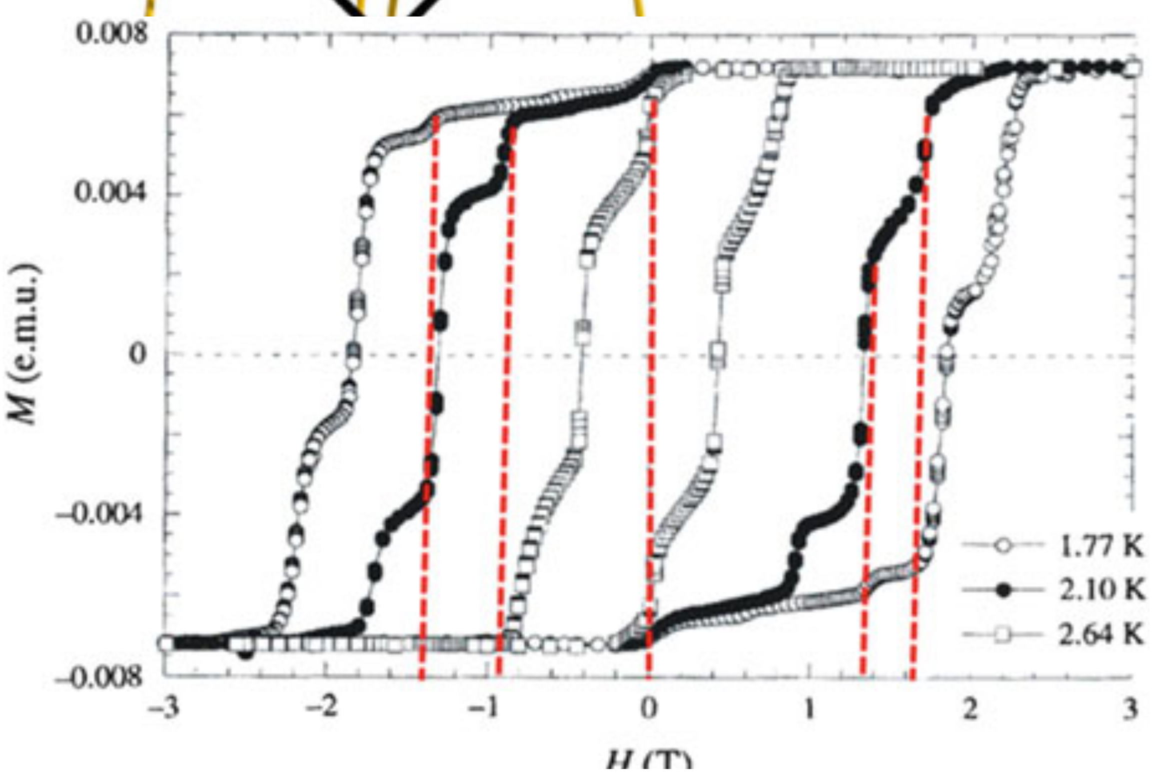
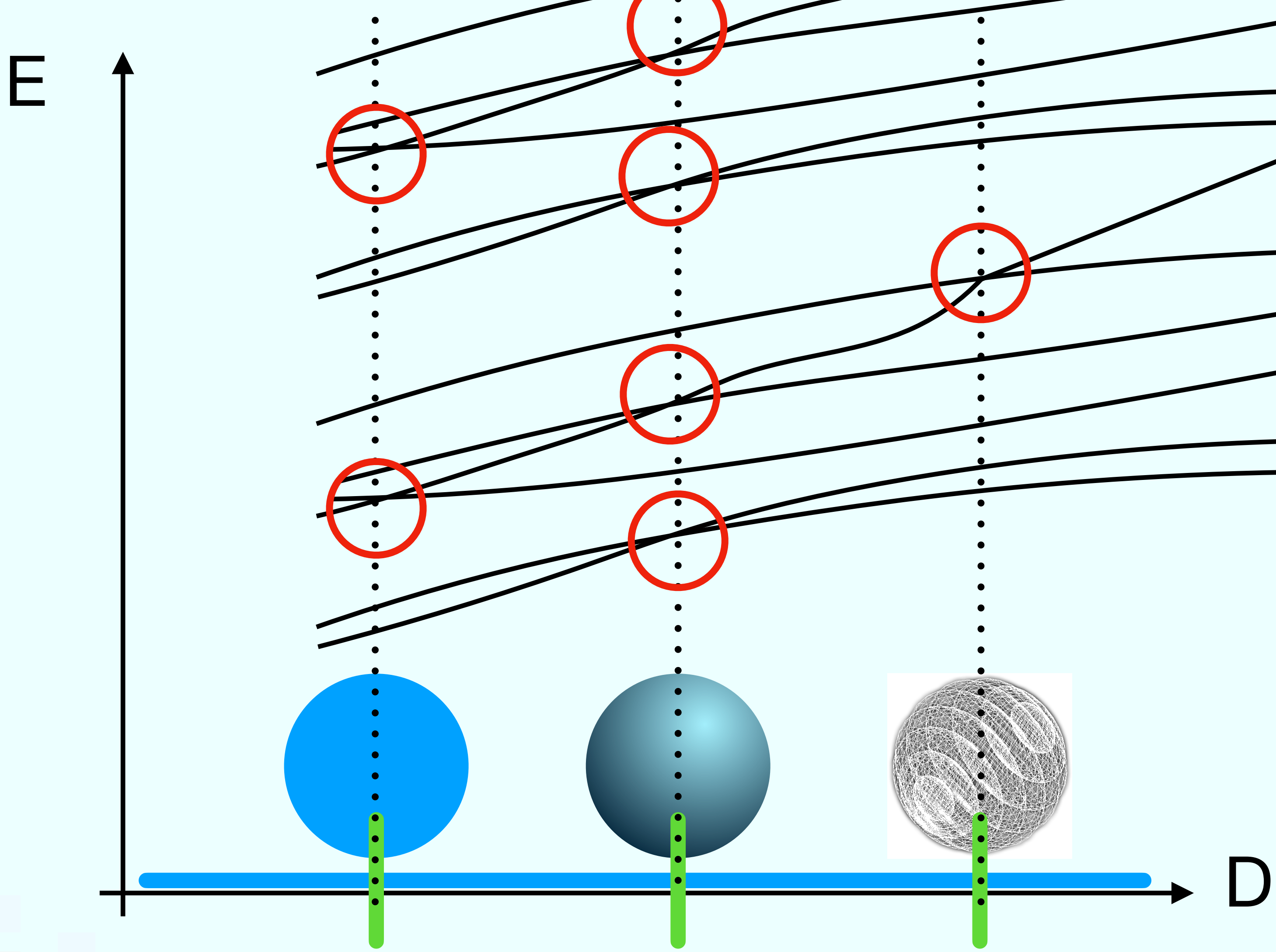




$$H = -U \sum_i (S_i^z)^2 - g S_i^z B$$



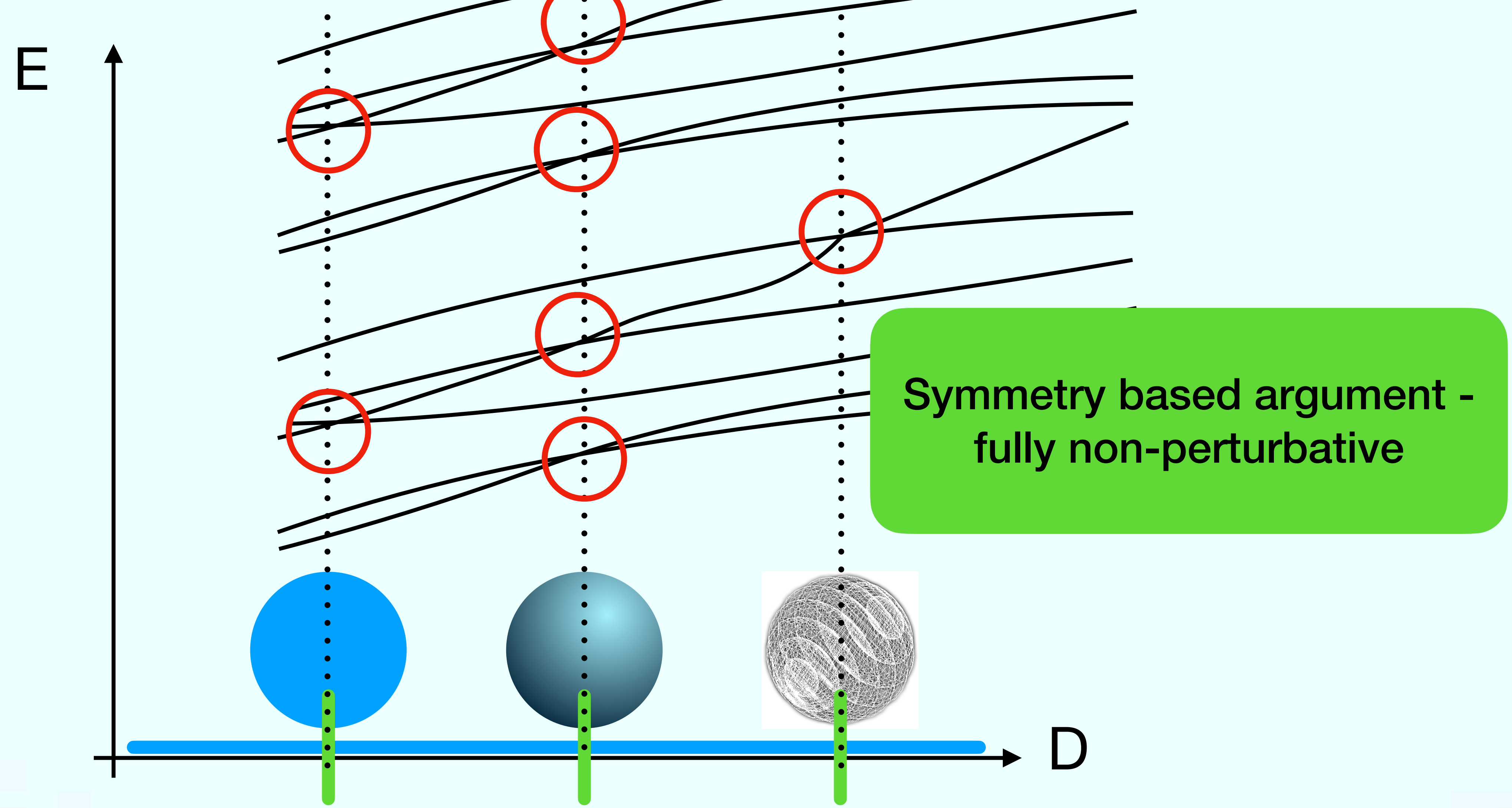
Tuning B field, degeneracies between the wells enhances quantum tunnelling, jumps in the hysteresis curve



Assume: spectrum continuity

Key result: the continued representation theory dictates some states drop out - “are evanescent” - *in pairs of equal energy*

This is a novel phenomena: we call it ‘evanescent-degeneracy’



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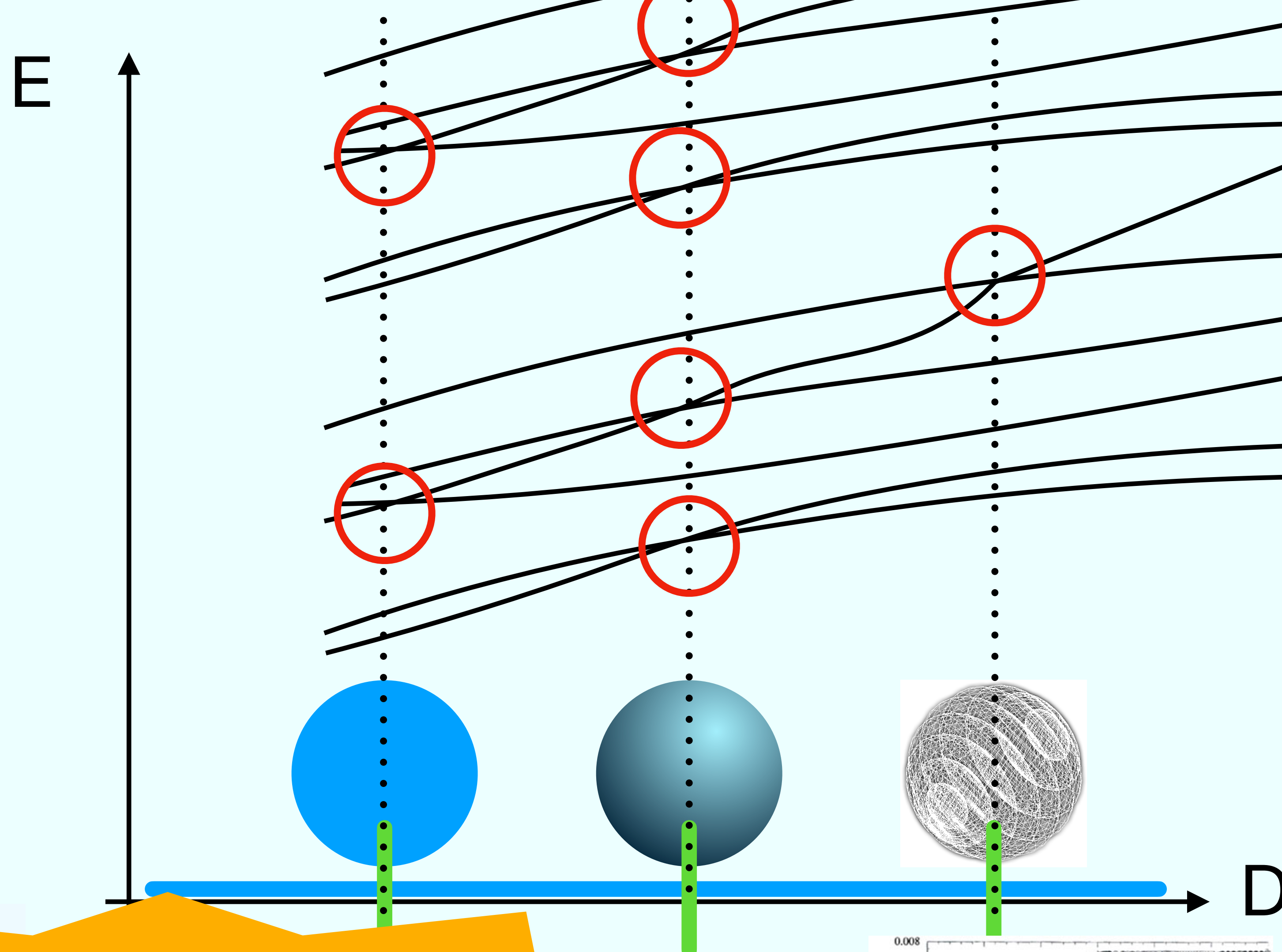
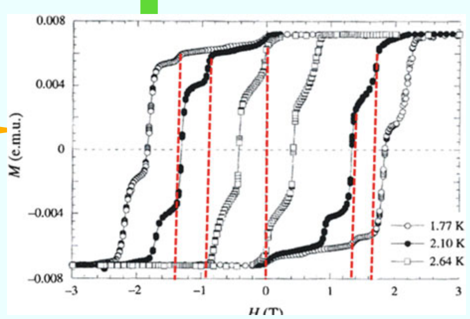
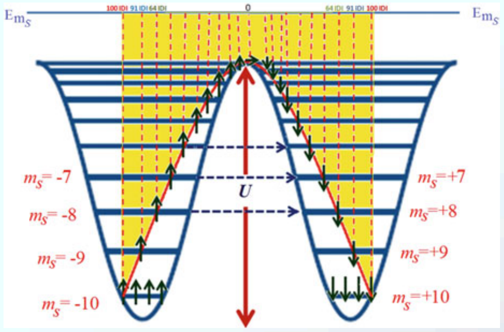


Degeneracy at isolated points of parameter N - when it is an integer

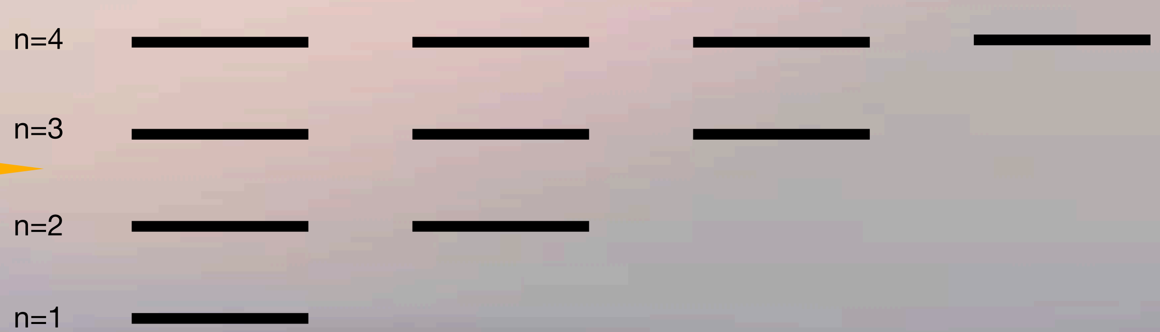




Accompanied by a major physical change: the theory becomes unitary

e.g. known existence of unitary islands with N=1,2,3 in d=3 via conformal bootstrap approach

Occurs between different irreps of O(N), a la Hydrogen spectrum



Adding spins

We all learn $$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$

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Writing this in terms of SU(2) representations, we can use Young diagram

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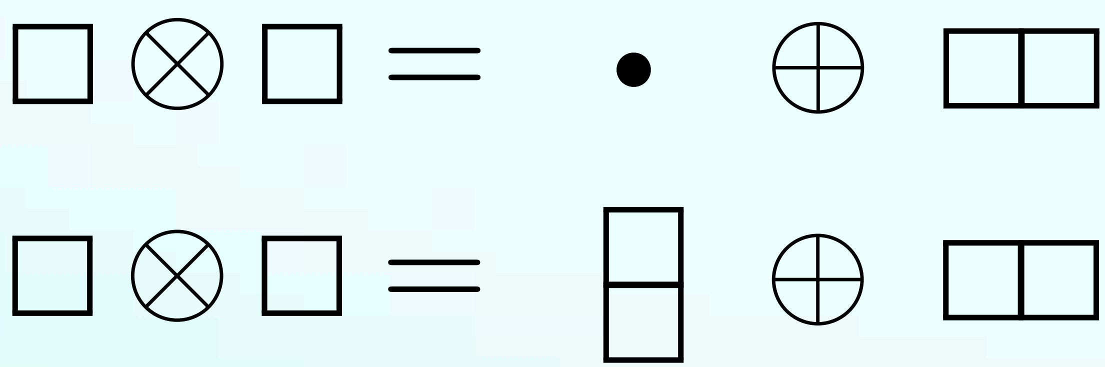
SU(2):

SU(3):



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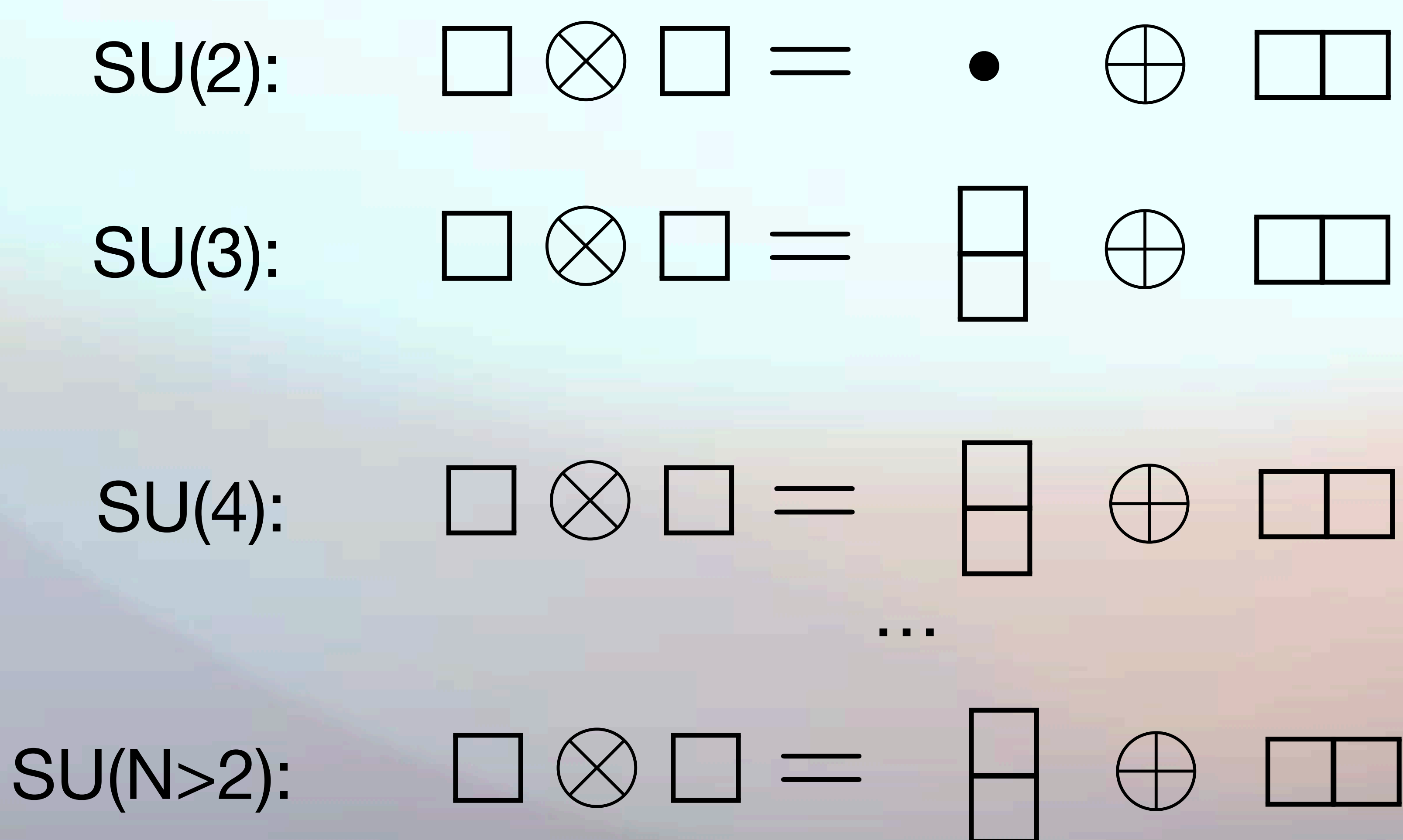
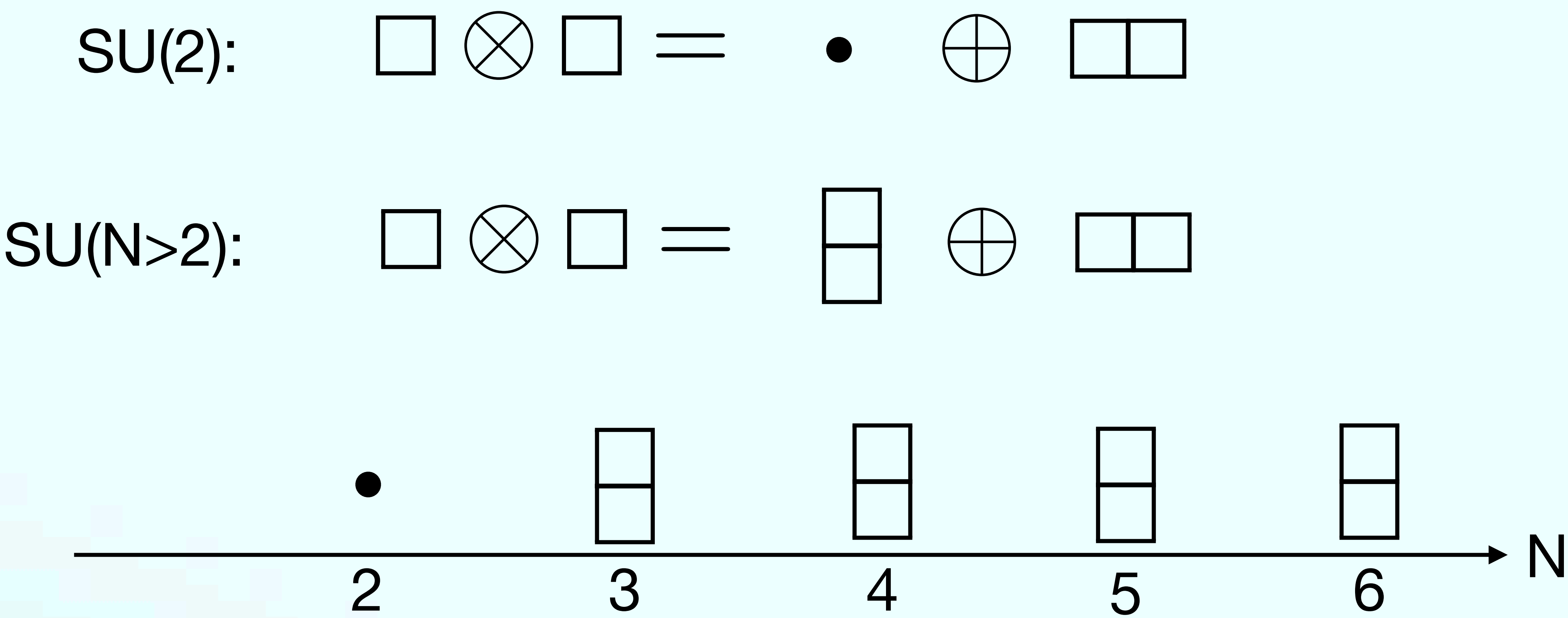


Adding spins

SU(2): $$\square \otimes \square = \bullet \oplus \square\!\square$$

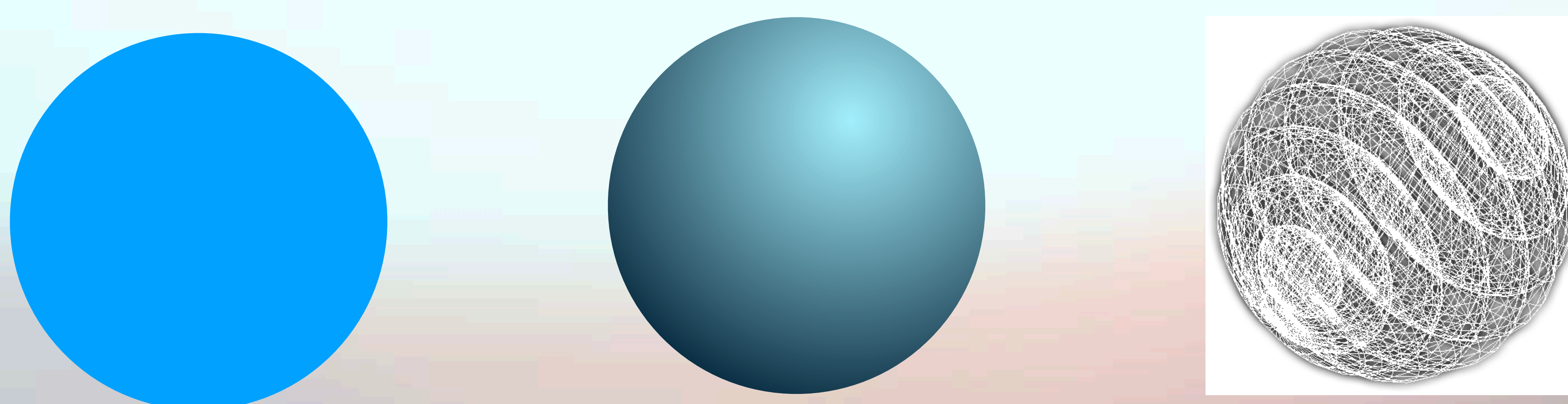
SU(N>2): $$\square \otimes \square = \begin{array}{c}\square\\[-4pt]\square\end{array} \oplus \square\!\square$$

Adding spins



c.f.

?



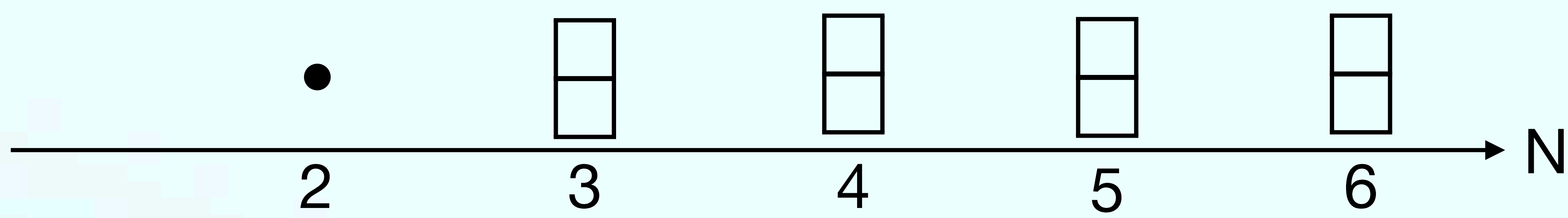
1D 2D 3D 4D

Adding spins

SU(2):

SU(N>2):

This is the right way to analytically continue.
Go to high enough N where we know what the representation is.
Supplement with rules at special N



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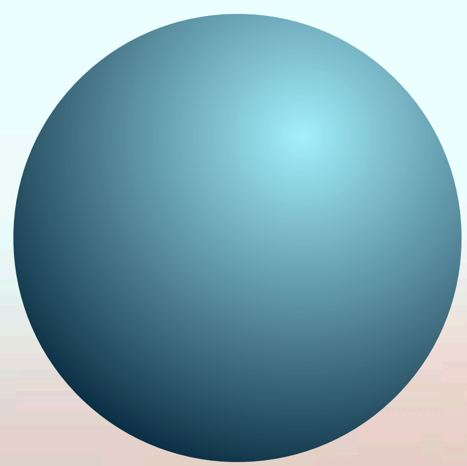

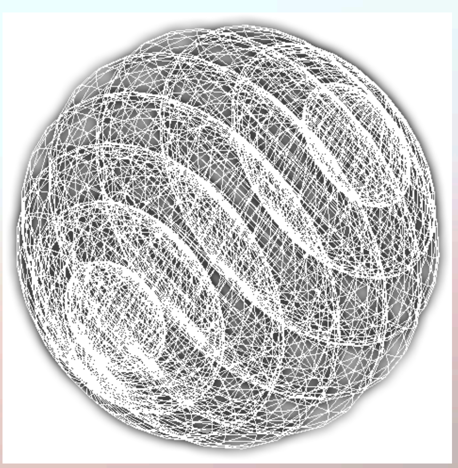

1D 2D 3D 4D

D

Analytic continuation for O(N)

Fixed integer N

$$R_1 \otimes R_2 = \sum_k c_k^{1,2}(N)\, R_k$$

Continued N

$$\bar{R}_1 \otimes \bar{R}_2 = \sum_k \bar{c}_k^{1,2}\, \bar{R}_k$$

With 'large N saturated' multiplicities and specialisation rules

$$\bar{c}_k^{1,2} = \lim_{N\to\infty} c_k^{1,2}(N)$$

$$\bar{R}_k \to \ ???$$

At 'too low' integer values of N

Analytic continuation for O(N)

Fixed integ

Continued N

These specialisation rules are known, but only relatively recently (in the context of the history of representation theory!)

K. Koike and I. Terada, Young-diagrammatic methods for the representation theory of the classical groups of type B_n, C_n, D_n, Journal of Algebra **107**, 466 (1987).

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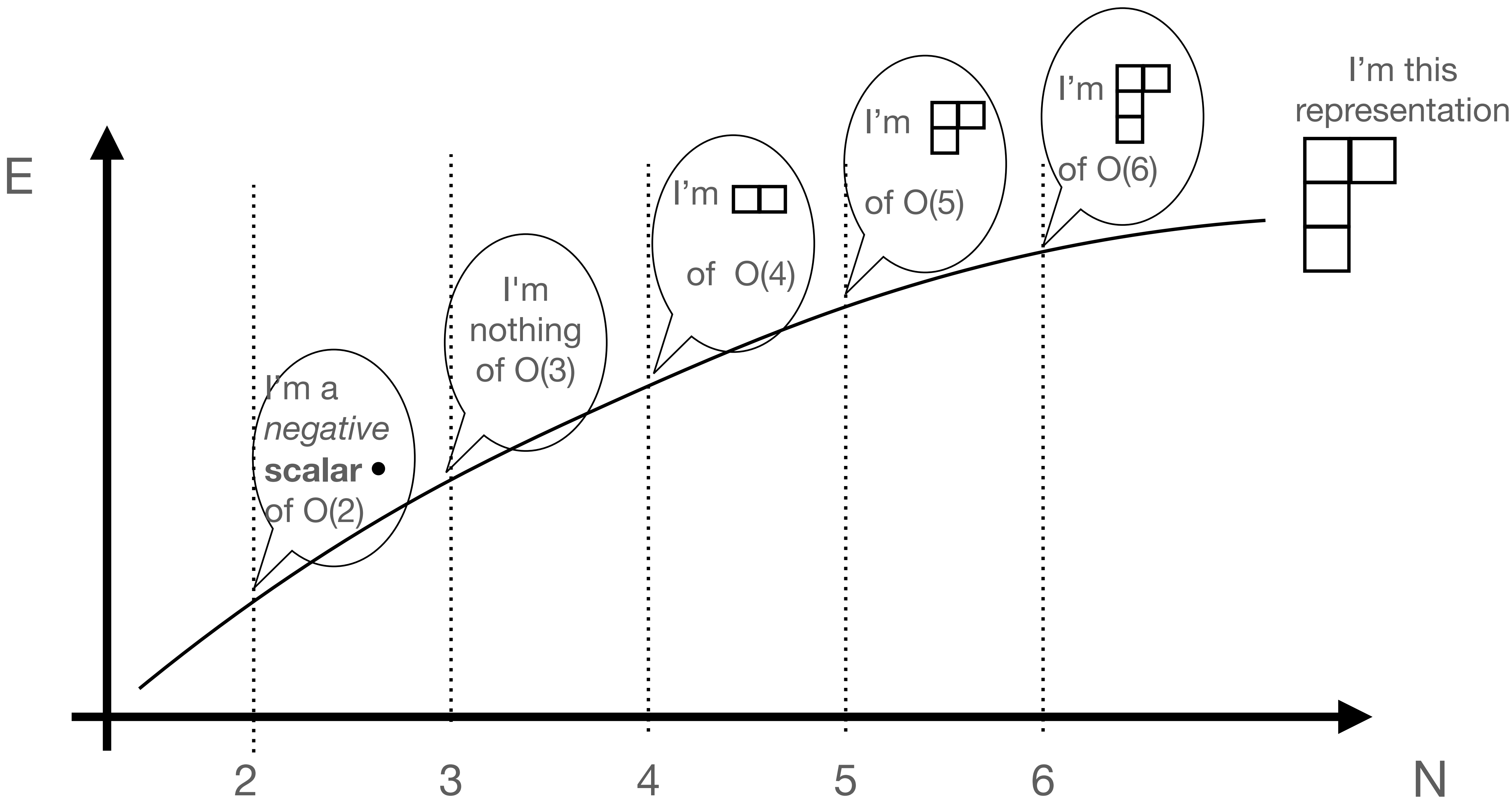
At 'too low' integer values of N

Example

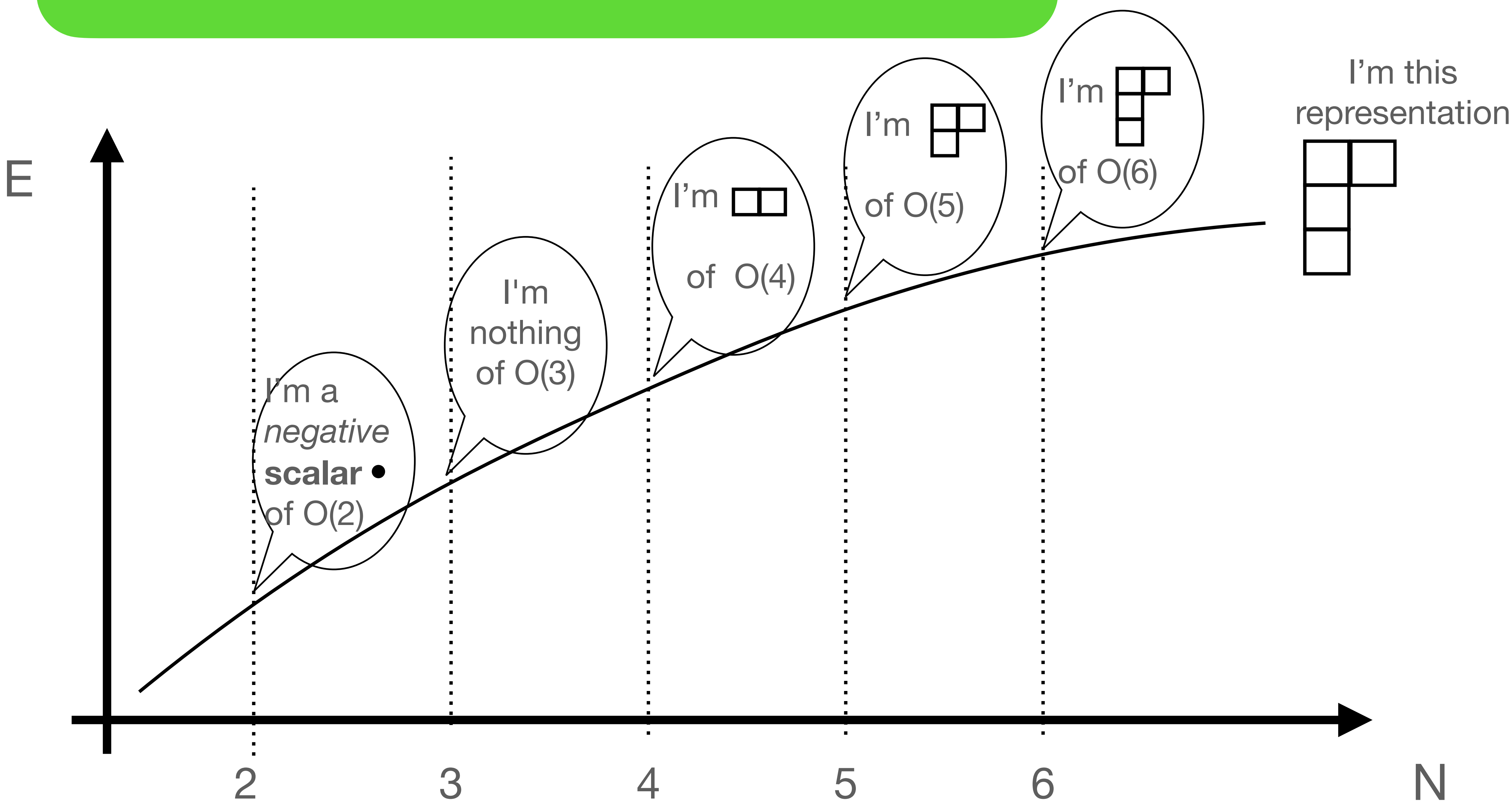
$$Z = \dim_{(2,1,1)} e^{-E_{(2,1,1)}(N)/T} + \ldots$$



Example

This is what is known in QFT as ‘evanescence’, and we can see there are a number of different types that can happen. (This had not been appreciated before)

+ . . .



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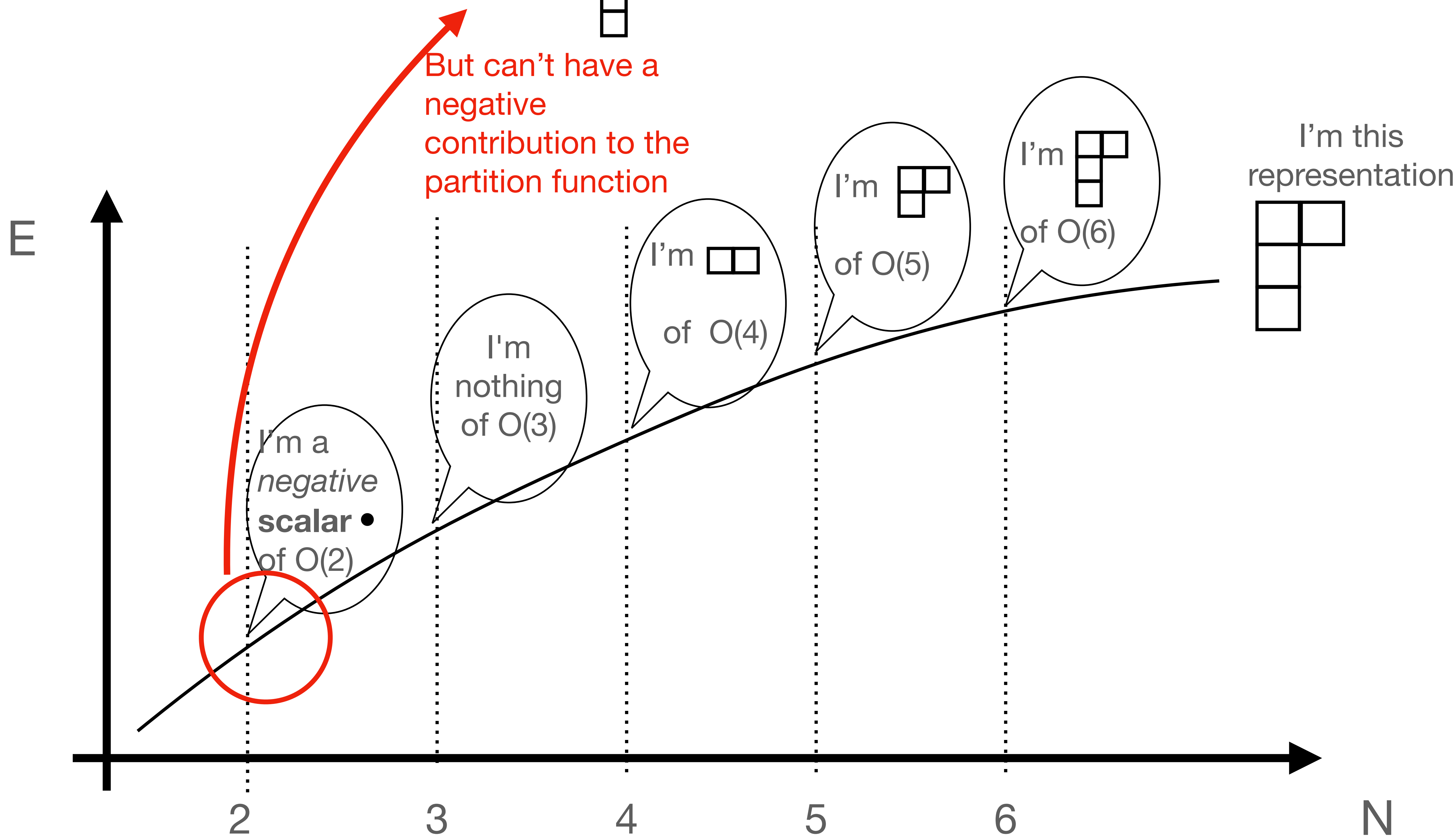

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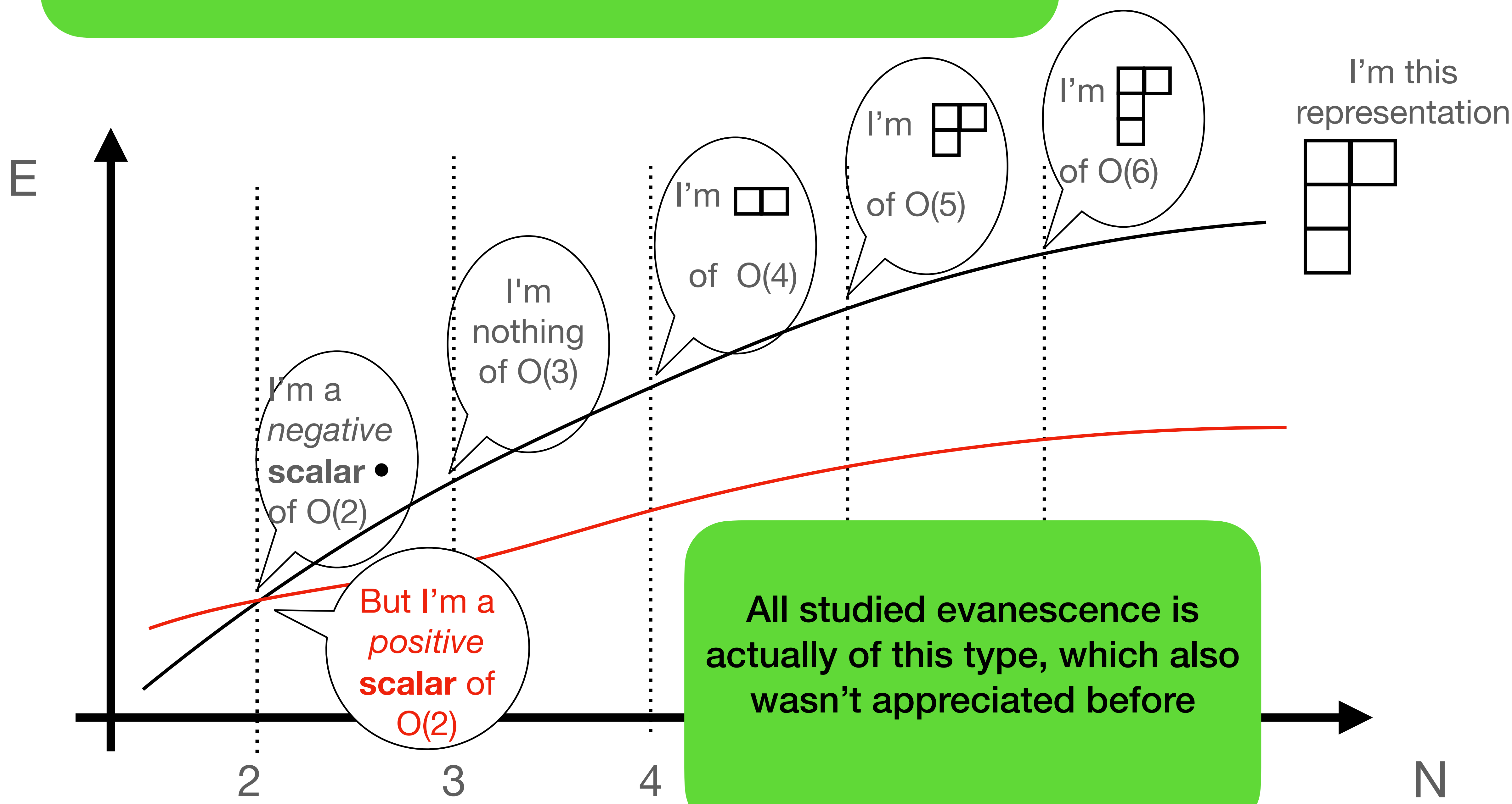

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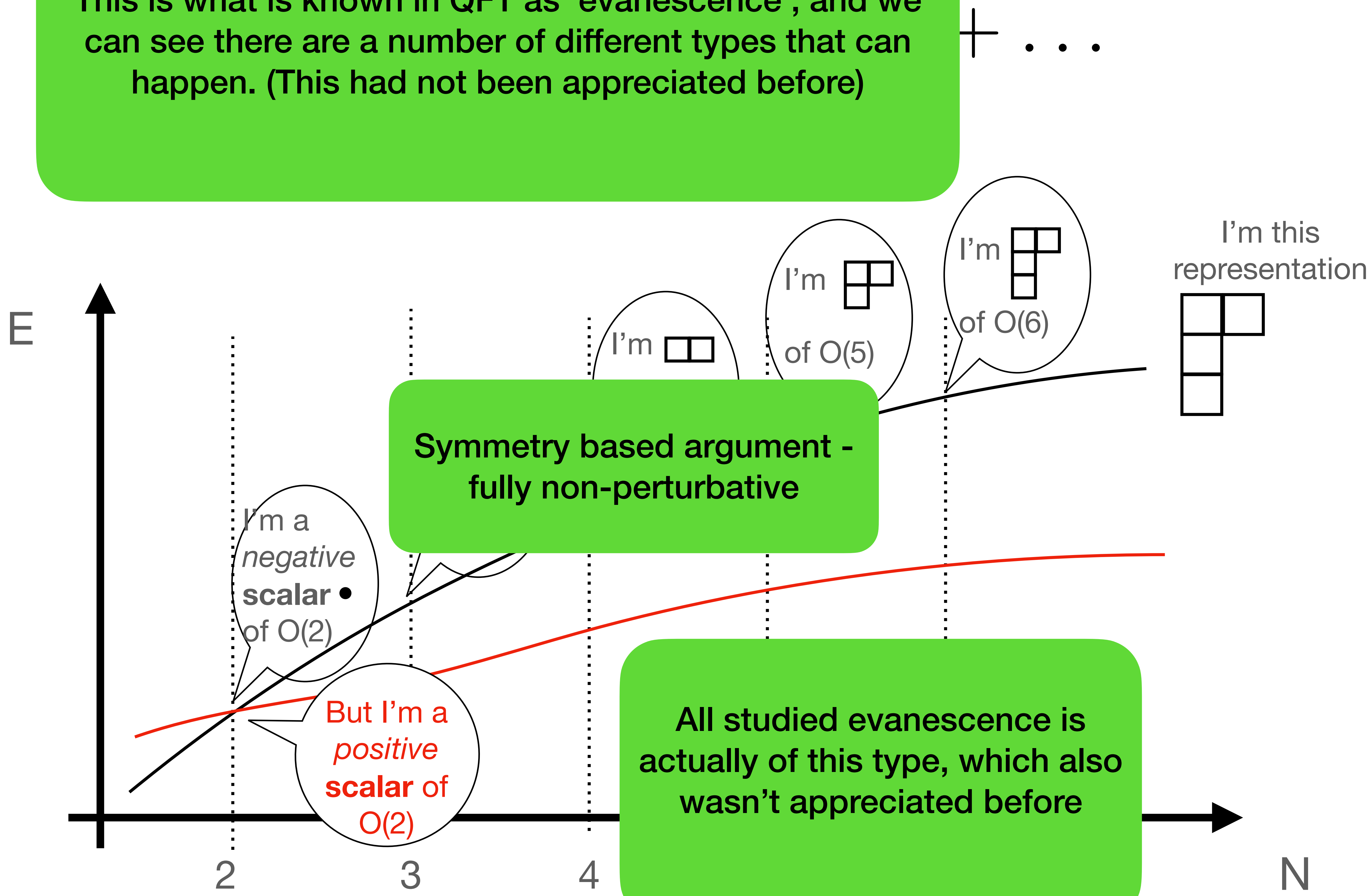

Symmetry based argument - fully non-perturbative

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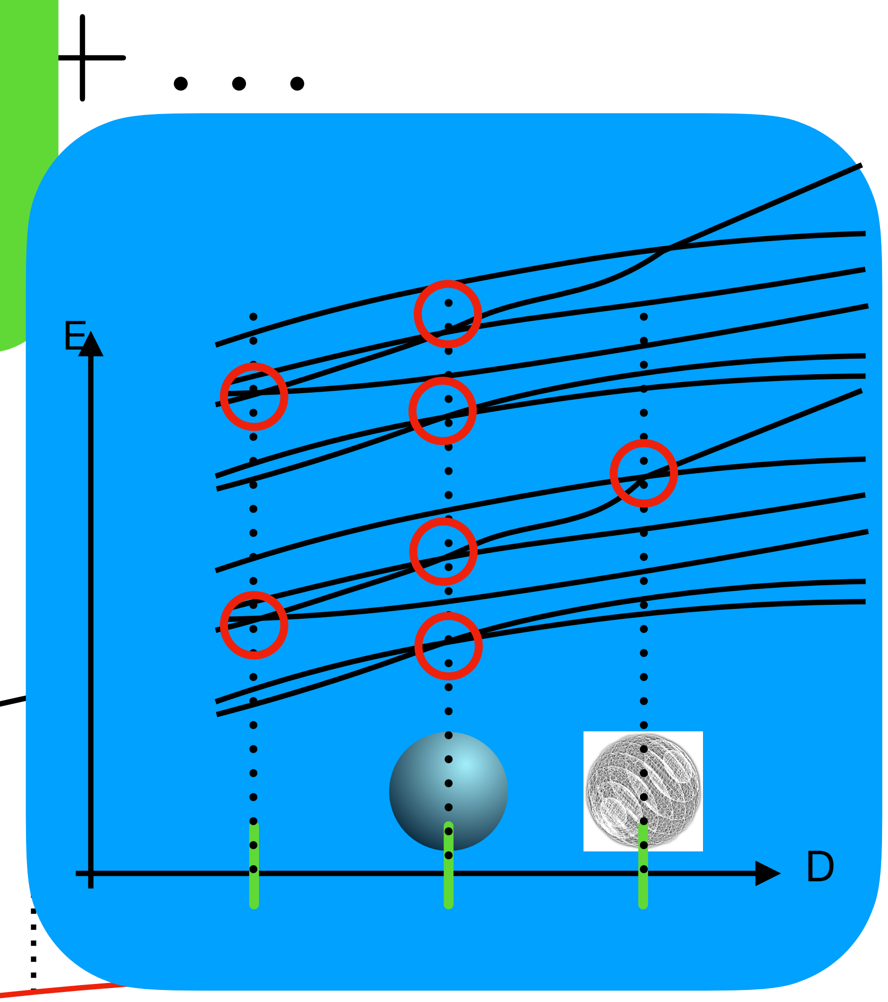

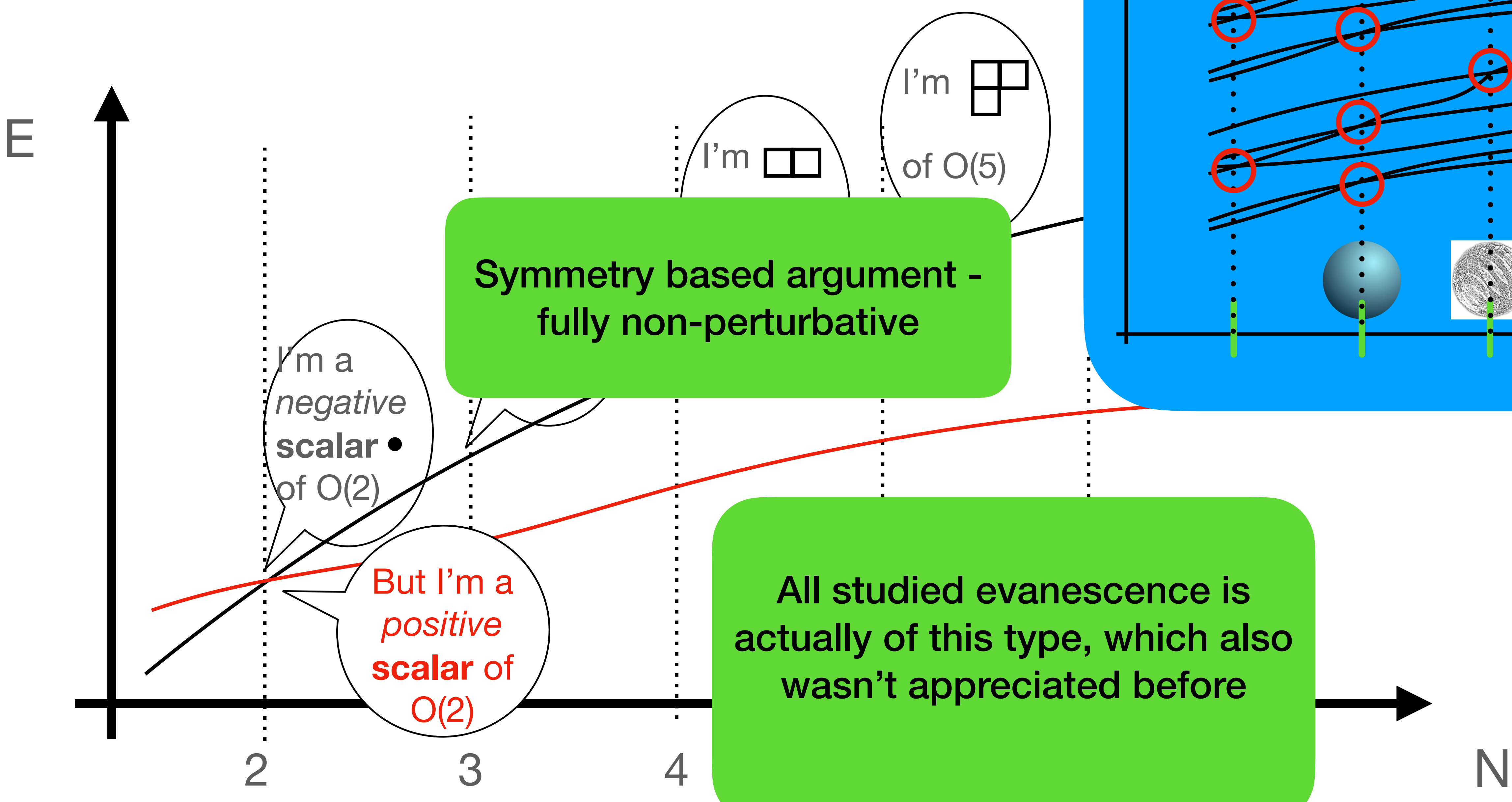

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Conclusion

It's quite a surprise that, given how ubiquitous O(N) symmetry is in physics, and how often we play around varying N, there were unnoticed physics consequences of symmetry

Learned something about dim reg

N is an interesting knob to start to investigate mechanisms of unitarity restoration/loss